Hudson Park High School

GRADE 12
MATHEMATICS
June Paper 1

Marks : 150

Time : 3 hours
Examiner : SLT

Date : June 2018
Moderator(s) : FRD and PHL

INSTRUCTIONS

1. Illegible work, in the opinion of the marker, will earn zero marks.
2. Number your answers clearly and accurately, exactly as they appear on the question paper.
3. **NB**
   - Leave 2 lines open between each of your answers.
   - Start each new QUESTION at the top of a new page.
4. **NB**
   - Fill in the details requested on the front of this Question Paper.
   - Detach the Answer Sheet for Question 7 and fill in the details requested on it.
   - Do not staple your Question Paper and Answer Pages together.
5. Employ relevant formulae and show all working out. Answers alone may not be awarded full marks.
6. (Non-programmable and non-graphical) Calculators may be used, unless their usage is specifically prohibited.
7. Round off answers to 2 decimal places, where necessary, unless instructed otherwise.
8. If (Euclidean) Geometric statements are made, reasons must be stated appropriately.
QUESTION 1

1.1. Solve for $x$ :
   1.1.1. $x^2 = 3x$ \hspace{1cm} (2)
   1.1.2. $2x = \frac{7}{x-3}$ \hspace{1cm} (4)
   1.1.3. $0 < 6x^2 - 7x - 3$ \hspace{1cm} (3)
   1.1.4. $2\sqrt{3-x} - 24 = 3x$ \hspace{1cm} (5)
   1.1.5. $6x^{\frac{4}{3}} + 7x^{\frac{2}{3}} - 24 = 0$ \hspace{1cm} (6)
   1.1.6. $4^{x-1} + 2^{2x} = 5\frac{3}{\sqrt{2}}$ \hspace{1cm} (5) (without the use of a calculator)

1.2. Solve for $x$ and $y$ :
   $y^2 - 2yx - x^2 = 31$ and $2y - x = 11$ \hspace{1cm} (6)

1.3. Discuss the nature of the roots of :
   $k^2x^2 - 4 = kx - x^2$

   where $k \in \mathbb{R}$. \hspace{1cm} (4)

1.4. Without the use of a calculator, simplify fully :

   \[
   \frac{\sqrt{27} - 5\sqrt{243}}{\sqrt{15}}
   \]

   leaving your answer with a rational denominator. \hspace{1cm} (5)
QUESTION 2

2.1. Given the arithmetic series:

\[(x + 1) + (-2x - 8) + (3 - x) + \cdots = 15476\]

2.1.1. Calculate the value of \(x\), showing that it will be \(-5\). \hfill (2)

2.1.2. Hence, determine the number of terms in the given arithmetic series. \hfill (6)

2.2. The \(88^{th}\) term of a quadratic sequence is \(-23647\). The general term of the first differences is given by \(-6n - 8\).

2.2.1. Write down the first three first differences of the quadratic sequence. \hfill (1)

2.2.2. Hence, determine an expression for the general term of the quadratic sequence. \hfill (4)

2.3. Given:

\[\frac{1}{6} + \frac{5}{42} + \frac{5}{56} + \frac{5}{72} + \cdots\]

For this series, determine:

2.3.1. \(S_1, S_2\) and \(S_3\) \hfill (1)

2.3.2. Hence, an expression for \(S_n\), the sum of the first \(n\)-terms of the series. \hfill (2)
QUESTION 3

3.1.1. Prove that the sum of the first $n$-terms of a geometric series whose first term is $a$ and having a common ratio of $r$, is given by:

$$S_n = \frac{a(r^n - 1)}{r - 1}$$  \hspace{1cm} (4)

3.1.2. Evaluate:

$$\sum_{k=3}^{14} 10 \left( \frac{3}{2} \right)^{2-k}$$  \hspace{1cm} (5)

3.2. Given the converging infinite geometric series:

$$\frac{1 - 2x}{3} + \frac{1 - 4x + 4x^2}{9} + \frac{(1 - 2x)^3}{27} + \ldots$$

3.2.1. Determine an expression for $r$, the common ratio of the series, in terms of $x$.  \hspace{1cm} (1)

3.2.2. Hence, determine the values of $x$ for which the series will converge.  \hspace{1cm} (4)

3.2.3. Now, if $x = -\frac{1}{2}$ determine $S_\infty$.  \hspace{1cm} (3)

[17]
4. The graphs of \( f \) and \( g \) have the defining equations of
\[
f(x) = -2(x - 1)^2 + 18 \quad \text{and} \quad g(x) = 2x + 4
\]
PQ is a vertical line whose \( x \) value lies between the \( x \) values of points A and D. B is the turning point of \( f \).

4.1. Determine the coordinates of:

4.1.1. A and C (label your answers clearly) \hspace{1cm} (3)

4.1.2. B \hspace{1cm} (2)

4.1.3. D \hspace{1cm} (5)

4.2. Calculate the maximum length of PQ. \hspace{1cm} (4)

4.3. Solve for \( x \):

4.3.1. \[ \frac{2x + 4}{-2(x - 1)^2 + 18} \leq 0 \] \hspace{1cm} (2)

4.3.2. \( x \cdot f(x) \geq 0 \) \hspace{1cm} (2)

4.4. Calculate the average gradient of \( f \) between \( x = -3 \) and \( x = 6 \). \hspace{1cm} (3)

4.5. Now, consider the inverse of \( f \), \( y = f^{-1}(x) \).

4.5.1. State the coordinates of the turning point of \( f^{-1} \). \hspace{1cm} (1)

4.5.2. Will \( f^{-1} \) be a function? \hspace{1cm} (1)
QUESTION 5

5. Given: \[ f(x) = \frac{7 - 5x}{x - 2} \]

5.1. Show that \( f \) can be written as: \[ f(x) = - \frac{3}{x - 2} - 5 \] (2)

5.2. State the range of \( f \). (1)

5.3. Sketch the graph of \( f \). All relevant details must be shown on the graph. (5) [8]

QUESTION 6

6.1. Determine the coordinates of \( A' \), the reflection of \( A(-7; 2) \) in the line \( y = -x + 4 \). (2)

6.2. The graph of \( g(x) = \frac{a}{x + p} + q \) has the following axes of symmetry:
\[ y = -x - 4 \] and \[ y = x + 3 \]
Calculate the values of \( p \) and \( q \). (3) [5]
7. The graph of \( f(x) = a \cdot 2^x \) is shown below:

![Graph of \( f(x) = a \cdot 2^x \)]

7.1. Determine the value of \( a \).  

7.2. On the axes given on the provided answer sheet, sketch:

7.2.1. \( y = x \) (accurately), and

7.2.2. \( f^{-1} \), the inverse of \( f \).

7.3. State the domain of \( f^{-1} \).

7.4. Determine the equation of \( f^{-1} \) in \( y \)-form.

7.5. If \( f \) were translated
   - 5 units vertically downwards, and
   - 4 units horizontally to the left
to become \( g \), state the equation of \( g \) in \( y \)-form.

[10]
QUESTION 8

8.1. How many months will it take for an investment of R 5 000 to grow into R 6 522.25 in an account that earns compound interest of 8 % per annum compounded monthly? (4)

8.2. A vehicle depreciates by a third of its value in 10 years, according to the diminishing balance method. Calculate the rate of depreciation. Leave your answer as a percentage. (4)

8.3. Convert a nominal interest rate of 7.5 % per annum compounded quarterly to an effective annual interest rate. Leave your answer as a percentage. (3)

[11]

QUESTION 9

9.1. When $3x^3 - 7ax^2 + 4x - 5$ is divided by $x + 2$, the remainder is 8. Calculate the value of $a$. (2)

9.2. Given: $f(x) = 30x^3 - x^2 - 61x + 12$

9.2.1. Use the factor theorem to show that $3x - 4$ is a factor of $f$. (2)

9.2.2. Hence, factorise $f$ fully. (3)

[7]
10.1. For two events, A and B:

\[
\begin{array}{c}
\text{A} \\
0.15 \\
x \\
2x \\
\text{B}
\end{array}
\]

it is known that
- \( P(\text{A only}) = 0.15 \)
- \( P(\text{A and B}) = x \)
- \( P(\text{B only}) = 2x \)
- \( P( (\text{A or B})' ) = 0.25 \)

10.1.1. Calculate the value of \( x \), showing that it will be 0.2. (2)

10.1.2. Are A and B independent events? Justify your answer showing all relevant working out. (4)

10.2. A bag contains 3 blue marbles and 8 red marbles. A marble is drawn from the bag, and not returned to the bag. A second marble is drawn from the bag.

10.2.1. Represent the given events in the form of a tree diagram. All relevant details must be shown on the diagram. (4)

10.2.2. Calculate the probability that two marbles of the same colour will be drawn. (3)

\[\text{TOTAL} \quad 150\]
INFORMATION SHEET

\[ x = -\frac{b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ A = P(1 + n) \quad A = P(1 - n) \quad A = P(1 - i)^n \quad A = P(1 + i)^n \]

\[ T_n = a + (n-1)d \quad S_n = \frac{n}{2} [2a + (n-1)d] \]

\[ T_n = ar^{n-1} \quad S_n = \frac{a(r^n - 1)}{r - 1} \quad r \neq 1 \quad S\infty = \frac{a}{1-r} \quad -1 < r < 1 \]

\[ F = x \left[ (1 + i)^{-1} \right] \]

\[ P = x \left[ \frac{1 - (1+i)^{-1}}{i} \right] \]

\[ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \]

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

\[ M \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \]

\[ y = mx + c \quad y - y_1 = m(x - x_1) \quad m = \frac{y_2 - y_1}{x_2 - x_1} \quad m = \tan \theta \]

\[(x - a)^2 + (y - b)^2 = r^2 \]

In \( \Delta ABC: \)
\[ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \]
\[ a^2 = b^2 + c^2 - 2bc \cos A \]
\[ \text{area} \Delta ABC = \frac{1}{2}ab \sin C \]

\[ \sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta \quad \sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta \]

\[ \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta \quad \cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta \]

\[ \cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha \]
\[ \sin 2\alpha = 2 \sin \alpha \cos \alpha \]

\[ \cos 2\alpha = \frac{1 - 2 \sin^2 \alpha}{2 \cos^2 \alpha - 1} \]

\[ \bar{x} = \frac{\sum x}{n} \quad \sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n} \]

\[ P(A) = \frac{n(A)}{n(S)} \quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \]

\[ \hat{y} = a + bx \quad b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2} \]

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ANSWER SHEET for QUESTION 7

7.

\[ f(x) = x^2 + 3 \]