INSTRUCTIONS

1. Illegible work, in the opinion of the marker, will earn zero marks.

2. Number your answers clearly and accurately, exactly as they appear on the question paper.

3. **NB**
   - Start each QUESTION at the top of a page.
   - Leave 2 lines open between each of your answers.

4. **NB** Fill in the details requested on the front of this Question Paper and hand in your submission in the following manner:
   - Question Paper (on top)
   - Answer Pages (below, in order)
   - *Do not staple the Question Paper and Answer Pages together.*

5. Employ relevant formulae and show all working out. Answers alone may not be awarded full marks.

6. (Non-programmable and non-graphical) Calculators may be used, unless their usage is specifically prohibited.

7. Round off answers to 2 decimal places, where necessary, unless instructed otherwise.

8. If (Euclidean) Geometric statements are made, reasons must be stated appropriately.
QUESTION 1 [ 36 marks ]

1.1. Solve for $x$:

1.1.1. $x^2 = 5x$ 3

1.1.2. $3x^2 - 4x - 12 = 0$ 3

1.1.3. $10x^{\frac{2}{3}} + 8x^{\frac{4}{3}} = 3$ 6

1.1.4. $0 \leq -x(6x + 5) + 4$ 3

1.1.5. $\frac{\sqrt{x}(2-x)}{2x(x-1)} \geq 0$ 2 (17)

1.2. Given: $6x^2 - 3y = 11x + 10$

$\frac{1}{3}x - y = \frac{16}{3}$

1.2.1. Solve for $x$ and $y$. 6

1.2.2. Interpret your answer to (1.2.1.) graphically. 2 (8)

1.3. Simplify fully: $\frac{2^{2015}}{2^{2017} - 3.2^{2012}}$ (3)

1.4. A quadratic equation was solved and its roots were found to be:

$x = \frac{3 \pm \sqrt{21 - 4k}}{5}$

where $k \in \mathbb{N}_0$. Determine the value(s) of $k$ for which the roots of the quadratic equation will be rational. (2)

1.5. The graph of $f$ has the following equation:

$y = x + \frac{1}{x}$

where $x \in \mathbb{R}$ and $x \neq 0$.

1.5.1. Write the given equation in standard form. 1

1.5.2. Now, determine the discriminant ($\Delta$) of the equation in (1.5.1.). 2

1.5.3. Hence, determine the range of $f$. 3 (6)
QUESTION 2 [ 28 marks ]

2.1. The 10th, 11th and 12th terms of an arithmetic sequence are:
\[ 2x + 3 ; 4x + 10 ; 10x - 3 \]

2.1.1. Calculate the value of \( x \), showing that it will be 5.  

2.1.2. Hence, determine \( T_{10} \) and \( T_{11} \).  

2.1.3. Now, calculate \( T_{500} \).  

2.2. Evaluate: \[ \sum_{k=7}^{95} (3 - 5k) \]  

2.3. Given: \[ \frac{3}{4} + \frac{1}{2} + \frac{1}{3} + \ldots \]

2.3.1. Calculate \( n \) if \( S_{\infty} - S_n = \frac{1024}{59049} \)  

2.3.2. You are given a tree of height 1.5 m as a gift. You plant it immediately. Each year you monitor it's growth and tabulate your results:

<table>
<thead>
<tr>
<th>Year</th>
<th>Growth (in metres)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.75</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
</tr>
<tr>
<td>3</td>
<td>0.3</td>
</tr>
</tbody>
</table>

What is the maximum height the tree will grow to?  

2.4. Given: \[ \frac{4}{16} ; -\frac{4}{8} ; -\frac{18}{4} ; -\frac{-38}{2} ; \ldots \]

Determine an expression for the \( n \)-th term of the sequence, \( T_n \).  

(6)
QUESTION 3 [ 19 marks ]

3.1. Given: \( f(x) = -2^x - 1 + 8 \)

3.1.1. Sketch the graph of \( f \), showing all relevant details on your diagram.  

3.1.2. State the range of \( f \).  

3.1.3. Calculate the average gradient of \( f \) between \( x = -1 \) and \( x = 1 \).  

3.1.4. \( f \) is reflected in the \( x \)-axis to become \( g \). Determine the equation of \( g \) in \( y \)-form.  

3.1.5. If \( h(x) = -16 \cdot 2^{x-1} + 10 \), give a detailed description of the transformation of \( f \) to \( h \). (13)

3.2. Given: \( l(x) = \log_{\frac{1}{4}} x \)

3.2.1. Sketch a rough graph of \( l \), showing all relevant details on your diagram.  

3.2.2. Solve for \( x \): \( \log_{\frac{1}{4}} x = 3 \)  

3.2.3. Hence, use your graph to solve for \( x \): \( \log_{\frac{1}{4}} x \geq 3 \) (6)
**QUESTION 4 [10 marks]**

4.1. Given: \( f(x) = \frac{8}{x+2} + 1 \)

4.1.1. Sketch the graph of \( f \), showing all relevant details on your diagram.

4.1.2. \( f \) is reflected in a certain line to become \( g \), where

\[ g(x) = \frac{8}{x+2} + 1 \]

State the possible equation(s) of that line.

4.1.3. If \( f \) is moved 5 units to the left, what will the new equation of \( f \) be?

4.2. For \( h(x) = \frac{5}{x+p} + q \) it is known that:

- the domain is: \( x \in \mathbb{R}, x \neq 4 \)
- one of the axes of symmetry is: \( y = -x + 7 \)

Determine the value of \( q \).
QUESTION 5 [ 17 marks ]

USE THE ANSWER SHEET PROVIDED

5.1. The following details are known about \( g(x) = -2x^2 + bx + c \):
   - the axis of symmetry is: \( x = 3 \)
   - \( g(-4) = 0 \)

Calculate the values of \( b \) and \( c \).

5.2. A sketch of \( f(x) = \frac{1}{4}(x + 5)^2 - 1 \) is shown below, where \( A \) is the turning point of \( f \):

For \( f \), determine the \( y \)-intercept, \( x \)-intercepts and coordinates of \( A \) and fill them in on the sketch.

On the same set of axes as \( f \), sketch \( f^{-1} \), the inverse of \( f \).

The intercepts and turning point of \( f^{-1} \) must be clearly labelled.

Determine the equation of \( f^{-1} \), the inverse of \( f \), in \( y \)-form.

\( f^{-1} \) is not a function.

Give a reason for this, with reference to \( f \).

State one way in which the domain of \( f \) be restricted so that \( f^{-1} \) would be a function.

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QUESTION 6 [ 12 marks ]

6.1. How many full years will it take an investment that is earning 6% interest per annum compounded monthly, to double in value? (4)

6.2. A vehicle depreciates by R 30 000 over a period of 5 years when the rate of depreciation, as calculated on the reducing balance method, is 7% per annum. Calculate the initial value of the vehicle. (4)

6.3. Convert a nominal interest rate of 6% per annum compounded monthly, to a nominal interest rate per annum compounded half-yearly. (4)
**QUESTION 7 [ 8 marks ]**

7.1. When \( f(x) = -2x^3 + ax^2 + 4 \) is divided by \((x + 3)\) the remainder is \(-14\). Calculate the value of \(a\). \(3\) marks

7.2. Given: \( f(x) = 6x^3 - 2x^2 + x + 35 \)

7.2.1. Use the factor theorem to show that \((3x + 5)\) is a factor of \(f\). \(2\) marks

7.2.2. Hence, determine the other (quadratic) factor of \(f\). \(3\) marks

**QUESTION 8 [ 20 marks ]**

8.1. If \( f(x) = \frac{3}{x} - 1 \), determine \(f'(x)\) from first principles. \(6\) marks

8.2. Determine:

8.2.1. \( \frac{dy}{dx} \), if \( y = \frac{x^2 + 5}{4 \cdot \sqrt[3]{x}} \) \(4\) marks

8.2.2. \( f''(x) \), if \( f(x) = x^{\frac{3}{2}} \left( x^{\frac{1}{2}} - x^{-\frac{3}{2}} \right) \) \(3\) marks

8.2.3. \( D_x \left[ \frac{8x^3 - 27}{2x - 3} \right] \) \(2\) marks

8.3. The tangent to \( f(x) = ax^2 + bx + 5 \) at the point \((-2; 9)\) is perpendicular to the line \(7y - 2x + 21 = 0\). Calculate the values of \(a\) and \(b\). \(5\) marks

- page 8 of 8 -
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ A = P(1 + n) \quad A = P(1 - n) \quad A = P(1 - i)^n \quad A = P(1 + i)^n \]

\[ T_n = a + (n - 1)d \quad S_n = \frac{n}{2} [2a + (n - 1)d] \]

\[ T_n = ar^{n-1} \quad S_n = \frac{a(r^n - 1)}{r - 1} \quad ; r \neq 1 \]

\[ F = x[(1 + i)^n - 1] \quad P = x[1 - (1 + i)^{-n}] \]

\[ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \]

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad M \left( \frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2} \right) \]

\[ y = mx + c \quad y - y_1 = m(x - x_1) \quad m = \frac{y_2 - y_1}{x_2 - x_1} \quad m = \tan \theta \]

\[ (x - a)^2 + (y - b)^2 = r^2 \]

In \( \triangle ABC \):

\[ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \]

\[ a^2 = b^2 + c^2 - 2bc \cos A \]

\[ \text{area } \triangle ABC = \frac{1}{2} ab \sin C \]

\[ \sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta \quad \sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta \]

\[ \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta \quad \cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta \]

\[ \cos 2\alpha = \frac{\cos^2 \alpha - \sin^2 \alpha}{2} \]

\[ \sin 2\alpha = 2\sin \alpha \cos \alpha \]

\[ \bar{x} = \frac{\sum fx}{n} \quad \sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n} \]

\[ P(A) = \frac{n(A)}{n(S)} \]

\[ P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \]

\[ \hat{y} = a + bx \quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2} \]
ANSWER PAGE FOR QUESTION 5

5.1.

5.2.