MATHEMATICS P2
JUNE 2014 – COMMON TEST
MEMORANDUM

NATIONAL SENIOR CERTIFICATE

GRADE 12

MARKS: 125

This memorandum consists of 11 pages.
### QUESTION 1

1.1.1 \[ AC^2 = (−1 − 4)^2 + (−4 − 1)^2 = 9 + 9 = 45 \]

\[ AC = \sqrt{18} \]

\[ = 3\sqrt{2} \quad (3) \]

✓ correct substitution into distance formula

✓ \( \sqrt{18} \)

✓ answer

1.1.2 \[ M \text{ is } \left( \frac{1 - 2}{2}; \frac{4 - 2}{2} \right) = \left( \frac{-1}{2}; 1 \right) \quad (2) \]

✓ answer

1.1.3 Gradient of AB \[ = \frac{-2 - 4}{-2 - 1} = \frac{-6}{-3} = 2 \]

\[ \therefore \text{gradient of } \perp \text{ bisector is } \frac{-1}{2} \]

(Obviously it passes through M)

\[ \left( \frac{-1}{2}; 1 \right) \]

\[ y = \frac{1}{2}x + c \]

Substitute \[ 1 = \left( \frac{-1}{2} \right) \left( \frac{-1}{2} \right) + c \]

\[ c = 1 - \frac{1}{4} \]

\[ = \frac{3}{4} \]

\[ \text{Equation is } y = -\frac{1}{2}x + \frac{3}{4} \quad (4) \]

✓ substitution of \( m \)

✓ and M into equation of line

✓ answer

1.2 \[ x^2 − 2x + 1 + y^2 + 2y + 1 = 2x − 2y \]

\[ x^2 − 4x + y^2 + 4y = −2 \]

\[ x^2 − 4x + 4 + y^2 + 4y + 4 = −2 + 4 + 4 \]

\[ (x − 2)^2 + (y + 2)^2 = 6 \]

\[ \therefore \text{centre is } (2; −2) \text{ and radius } = \sqrt{6} \quad (6) \]

✓ \( x^2 − 4x + 4 \)

✓ \( y^2 + 4y + 4 \)

✓ \( (x − 2)^2 \)

✓ \( (y + 2)^2 \)

✓ answer: centre

✓ answer: radius

[15]
### QUESTION 2

#### 2.1.1

Since $JK \parallel LM$, $M_{JK} = M_{LM}$

$$\begin{align*}
-4 - 1 &= 0 - 2 \\
p + 2 &= 5 - 3 \\
-2(p + 2) &= (-5)(2) \\
-2p - 4 &= -10 \\
-2p &= -6 \\
x &= 3
\end{align*}$$

- equating gradients
- substitution in each gradient
- simplification

#### 2.1.2

$$\begin{align*}
JK^2 &= (3 + 2)^2 + (-4 - 1)^2 = 50 \\
LM^2 &= (2 - 0)^2 + (3 - 5)^2 = 8 \\
JK : LM &= \sqrt{50} : \sqrt{8} = 5\sqrt{2} : 2\sqrt{2} \\
&= 5 : 2
\end{align*}$$

- correct substitution into distance form.
- $\sqrt{50}$
- $\sqrt{8}$
- ratio
- answer

#### 2.1.3

Diagonals of a parallelogram bisect each other

Midpoint of JL : $\left( \frac{3}{2}, \frac{1}{2} \right)$

This is the same midpoint for MQ:

Thus $\frac{x + 3}{2} = \frac{3}{2}$; $\frac{y - 2}{2} = -\frac{1}{2}$

\[
\therefore x = 0 \quad \therefore y = -1
\]

Therefore Q is Q(0; -1)

- $\left( \frac{3}{2}, \frac{1}{2} \right)$
- $\frac{x + 3}{2} = \frac{3}{2}$
- $\frac{y - 2}{2} = -\frac{1}{2}$
- answer OR answer only (if done by inspection) - full marks

#### 2.1.4

Since the x coordinates of K and M are both 3, it follows the equation of KM is $x = 3$.

- answer

#### 2.1.5

90º since KM is a vertical line

- answer

#### 2.1.6

For collinearity; $m_{JR} = m_{JL}$

$$m_{JR} = \frac{k - 1}{1 - (-2)} = \frac{0 - 1}{5 - (2)}$$

$$\begin{align*}
&= \frac{k - 1}{3} = -\frac{1}{7} \\
&\therefore k - 1 = -\frac{3}{7} \\
&\therefore k = \frac{-3}{7} + 1 \\
k &= \frac{4}{7}
\end{align*}$$

- equating: $m_{JR} = m_{JL}$
- $m_{JR} = \frac{k - 1}{1 - (-2)}$
- simplification
- answer
### 2.2.1

<table>
<thead>
<tr>
<th><strong>Q (x; 2) … (radius QR ⊥ tangent)</strong></th>
<th><strong>(5)</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Substitute (x; 2) in (3x + 4y + 7 = 0):</td>
<td></td>
</tr>
<tr>
<td>(3x + 8 + 7 = 0)</td>
<td></td>
</tr>
<tr>
<td>(x = -5)</td>
<td></td>
</tr>
<tr>
<td>(\therefore \ Q (-5; 2))</td>
<td></td>
</tr>
<tr>
<td>Radius = (QR = 0 - (-5) = 5)</td>
<td></td>
</tr>
<tr>
<td>(\therefore ) Equation is ((x + 5)^2 + (y - 2)^2 = 25)</td>
<td>(\checkmark)</td>
</tr>
</tbody>
</table>

### 2.2.2

| **QR = 5 units** | **(1)** |
| **d = 2 x radius** |         |
| **\(\therefore \ WZ = 10\) units** | \(\checkmark\) |

\[\text{[27]}\]
QUESTION 3

3.1 \[
\sin 15^\circ = \sin (45^\circ - 30^\circ) \\
= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ \\
= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2} \\
= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} \\
= \frac{\sqrt{3} - 1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\
= \frac{\sqrt{6} - \sqrt{2}}{4} \tag{3}
\]

OR

\[
\sin 15^\circ = \cos 75^\circ \\
= \cos (45^\circ + 30^\circ) \\
= \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ \\
= \left( \frac{1}{\sqrt{2}} \right) \left( \frac{\sqrt{3}}{2} \right) - \left( \frac{1}{\sqrt{2}} \right) \left( \frac{1}{2} \right) \\
= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} \\
= \frac{\sqrt{3} - 1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\
= \frac{\sqrt{6} - \sqrt{2}}{4} \tag{3}
\]

3.2 \[
\frac{\tan (180^\circ + \theta) \cos (360^\circ - \theta)}{\sin (180^\circ - \theta) \cos (90^\circ + \theta) + \cos (540^\circ + \theta) \cos (-\theta)} \\
= \frac{\tan \theta \cdot (\cos \theta)}{\sin \theta \cdot (-\sin \theta) - \cos \theta \cdot \cos \theta} \\
= \frac{\sin \theta}{\cos \theta} \times \cos \theta \\
= -\sin^2 \theta - \cos^2 \theta \\
= -\sin^2 \theta + \cos^2 \theta \\
= -\sin \theta \tag{9}
\]

For each reduction:
\checkmark \tan \theta \checkmark \cos \theta
\checkmark \sin \theta \checkmark -\sin \theta
\checkmark \cos \theta \checkmark \cos \theta
\checkmark \tan \theta = \frac{\sin \theta}{\cos \theta}
\checkmark \sin^2 \theta + \cos^2 \theta
\checkmark \text{answer}
3.3 \[
\sin \left(45^\circ + x\right) \cdot \sin \left(45^\circ - x\right) = \frac{1}{2} \cos 2x
\]

LHS
\[
= \sin \left(45^\circ + x\right) \cdot \sin \left(45^\circ - x\right)
\]
\[
= \left(\sin 45^\circ \cos x + \cos 45^\circ \sin x\right) \sin 45^\circ \cos x - \sin x \cos 45^\circ
\]
\[
= \left(\frac{1}{\sqrt{2}} \cos x + \frac{1}{\sqrt{2}} \sin x\right) \left(\frac{1}{\sqrt{2}} \cos x - \frac{1}{\sqrt{2}} \sin x\right)
\]
\[
= \frac{1}{2} \cos^2 x - \frac{1}{2} \sin^2 x
\]
\[
= \frac{1}{2} \left(\cos^2 x - \sin^2 x\right)
\]
\[
= \frac{1}{2} \cos 2x
\]
\[
= \text{RHS}
\]

(5)

3.4 \[
\frac{\sin 33^\circ}{\sin 11^\circ} = \frac{\cos 33^\circ}{\cos 11^\circ}
\]
\[
= \frac{\sin 33^\circ \cdot \cos 11^\circ - \cos 33^\circ \cdot \sin 11^\circ}{\sin 11^\circ \cdot \cos 11^\circ}
\]
\[
= \frac{\sin \left(33^\circ - 11^\circ\right)}{\sin 11^\circ \cos 11^\circ}
\]
\[
= \frac{\sin 22^\circ}{\sin 11^\circ \cos 11^\circ}
\]
\[
= \frac{2 \sin 11^\circ \cos 11^\circ}{\sin 11^\circ \cos 11^\circ}
\]
\[
= 2
\]

(6)

3.5 \[
\tan \frac{3x}{\tan 24^\circ} = 1
\]
\[
\tan 3x = \tan 24^\circ
\]
\[
3x = 24^\circ + k \cdot 180^\circ
\]
\[
\therefore x = 8^\circ + k \cdot 60^\circ, k \in \mathbb{Z}
\]

(5)

(28)
# QUESTION 4

### 4.1

Constr: Draw $FQ \perp HG$ produced

\[
\sin FGH = \sin(180^\circ - FQG) = \sin FQG
\]

\[
\frac{FQ}{h} = \sin G
\]

\[
\therefore FQ = h \sin G
\]

Also \[
\frac{FQ}{g} = \sin H
\]

\[
\therefore g = \frac{FQ}{\sin H}
\]

\[
\therefore \sin G = \frac{g \sin H}{h}
\]

\[ (5) \]

### 4.2.1

\[
\frac{PW}{PQ} = \tan \alpha
\]

\[ \therefore PW = PQ \tan \alpha \]

\[ (2) \]

### 4.2.2

$QPR = 180^\circ - (x + y)$ and

\[
\sin QPR = \sin \left[180^\circ - (x + y)\right] = \sin (x + y)
\]

In $\triangle PQR$, by the sine rule

\[
\frac{PR}{\sin Q} = \frac{QR}{\sin QPR}
\]

\[
PR = \frac{15}{\sin y} \sin [180^\circ - (x + y)]
\]

\[
\therefore PR = \frac{15 \sin y}{\sin (x + y)}
\]

\[ (4) \]
4.2.3 \[ PW = \frac{PR\tan\alpha}{\tan\beta} \] from 3.3.1

\[
PW = \frac{15 \sin y}{\sin (x+y)} \cdot \tan \alpha
\]

\[
= \frac{15 \sin y}{\sin (y + y)} \cdot \tan \alpha
\]

\[
= \frac{15 \sin y}{\sin 2y} \cdot \tan \alpha
\]

\[
= \frac{15 \sin y}{2 \sin y \cdot \cos y} \cdot \tan \alpha
\]

\[
= \frac{7.5}{\cos y} \cdot \tan \alpha
\]

\[
\therefore PW = 7.5 \frac{\tan \alpha}{\cos y}
\]

\[\checkmark PW = \frac{15 \sin y}{\sin (x+y)} \cdot \tan \alpha\]

\[\checkmark \text{simplification}\]

\[\checkmark \sin 2y = 2 \sin y \cdot \cos y\]

(3) [14]
**QUESTION 5**

<table>
<thead>
<tr>
<th>5.1</th>
<th>Supplementary ✓</th>
<th>(1)</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.2.1</td>
<td>It is the angle between tangent and radius. ✓</td>
<td>(1)</td>
<td>Answer</td>
</tr>
</tbody>
</table>
| 5.2.2 | \( \hat{S}_1 + \hat{S}_4 = 90^\circ \) ... Tan \( \perp \) radius  
\( \hat{N}_1 + \hat{N}_2 = 90^\circ \) ✓ ... Tan \( \perp \) radius ✓  
\( \hat{S}_1 + \hat{S}_4 + \hat{N}_1 + \hat{N}_2 = 90^\circ + 90^\circ \) ✓  
\( = 180^\circ \) ✓  
\( \therefore \) RNOS is a cyclic quadrilateral ... opp \( \angle \) quad supplementary ✓ | (4) | Statement Reason |
| 5.2.3 | \( \hat{S}_1 = x \)  
\( S_1 = \hat{N}_2 = x \) ✓ ... Tan chord theorem ✓  
\( \hat{N}_2 = S_3 = x \) ✓ ... base \( \angle \)'s of isosceles \( \triangle \) OSN ✓  
\( \hat{S}_3 = \hat{R}_2 = x \) ✓ ... \( \angle \)'s in same segment ✓  
\( \hat{N}_2 = \hat{R}_1 = x \) ✓ ... \( \angle \)'s in same segment. ✓ | (8) | Statement Reason |
| 5.2.4 | \( \hat{O}_1 + \hat{O}_2 + \hat{N}_2 + \hat{S}_3 = 180^\circ \) ... sum of \( \angle \)'s of \( \triangle \) OSN ✓  
But \( \hat{S}_3 = \hat{N}_2 \) ... \( \angle \) opp. equal sides ✓  
\( \therefore \hat{O}_1 + \hat{O}_2 + 2\hat{S}_3 = 180^\circ \)  
\( \hat{O}_1 + \hat{O}_2 + \hat{O}_3 = 180^\circ \) ... \( \angle \)'s on a straight line ✓  
\( \therefore \hat{O}_1 + \hat{O}_2 + 2 \hat{S}_3 = \hat{O}_1 + \hat{O}_2 + \hat{O}_3 \)  
\( 2 \hat{S}_3 = \hat{O}_3 \) ✓  
\( \therefore \hat{S}_3 = \frac{1}{2} \hat{O}_3 \) | (4) | Statement with reason |

**OR**

\( \hat{O}_3 = \hat{S}_R \hat{N} \) ... ext. \( \angle \) 's of cyclic quad. ✓  
\( \hat{R}_2 = \hat{S}_3 \) ... \( \angle \)'s in the same segment ✓  
but \( \hat{S}_R = \hat{N} \) ... radii of a circle  
\( \therefore \hat{R}_1 = \hat{R}_2 \) ... chords ; = \( \angle \) ✓  
\( \therefore \hat{S}_3 = \frac{1}{2} \hat{O}_3 \) | (4) | Statement Reason |

[18]
**QUESTION 6**

6.1 Given: \( \triangle ABC \) with \( D \) on \( AB \) and \( E \) on \( AC \) such that \( DE \parallel BC \)

![Diagram](image)

RTP: \[
\frac{AD}{DB} = \frac{AE}{EC}
\]

Construction: Join \( DC \) and \( BE \)

Proof:

\[
\frac{\text{area } \triangle ADE}{\text{area } BDE} = \frac{1}{2} \cdot \frac{AD \cdot h_1}{DB \cdot h_2} = \frac{AD}{DB} \quad \therefore \text{areas of } \triangle \text{'s with the same height and common vertex are in the same ratio as their bases}
\]

\[
\frac{\text{area } \triangle ADE}{\text{area } DEC} = \frac{1}{2} \cdot \frac{AE \cdot h_2}{EC \cdot h_2} = \frac{AE}{EC} \quad \therefore \text{same base, same parallel lines}
\]

Thus \[
\frac{AD}{DB} = \frac{AE}{EC} \quad (6)
\]

6.2.1 In \( \triangle PQM \), \( GH \parallel QT \)

\[
\frac{QH}{HM} = \frac{GP}{GM} \quad \checkmark \text{(line parallel to one side of a } \triangle \text{ OR Prop Th: } GH \parallel PQ \checkmark)
\]

\[
= \frac{1}{2} \quad (3)
\]

6.2.2 \[
QH = k; \quad HM = 2k \quad \therefore RM = 3k \quad \checkmark
\]

\[
\frac{RG}{RT} = \frac{RH}{RQ} \quad \checkmark \text{(line parallel to one side of a } \triangle \text{ OR Prop Th: } GH \parallel PQ \checkmark)
\]

\[
= \frac{5k}{6k} \quad \checkmark
\]

\[
= \frac{5}{6} \quad (5)
\]

RM = 3k

Statement Reason

Substitution

Answer
| 6.3.1 | Let \( \hat{Z}_2 = x = \alpha \) \( \ldots \) Tan chord theorem \( \checkmark \) |
| Statement Reason |
| Then \( \hat{A}\hat{B}X = 90^\circ - \alpha \ldots \) sum of \( \angle \)'s of \( \Delta \) \( ABP \) \( \checkmark \) |
| Statement with reason |
| But \( \hat{Z}_1 = \hat{A}\hat{B}P \) |
| = \( 90 - \alpha \ldots \angle \)'s opposite equal sides: \( AZ = AB \) \( \checkmark \) |
| Statement with reason |
| \( \hat{Z}_1 + Z_x = \alpha + 90^\circ - \alpha \ldots \) adj. \( \angle \)'s on a straight line \( \checkmark \) |
| Statement with reason |
| Thus \( \hat{Z}_3 = 90^\circ \) |
| (5) |

| 6.3.2 | In \( \Delta AYZ \) and \( \Delta AZX \) |
| Statement with reason |
| 1. \( \hat{Z}_2 = \hat{X} \ldots \) Tan chord theorem \( \checkmark \) |
| Statement |
| 2. \( \hat{A}_2 = \hat{A}_2 \ldots \) common \( \checkmark \) |
| Statement |
| 3. \( A\hat{Y}Z = A\hat{Z}X \) (remaining angles) |
| \( \therefore \Delta AYZ \parallel \Delta AZX \ \angle, \angle, \angle \) \( \checkmark \) |
| (3) |

| 6.3.3 | \( \therefore \frac{AZ}{AY} = \frac{AX}{AZ} \) \( \Delta \)'s \( \parallel \) sides in proportion \( \checkmark \) |
| (1) |
| \( \therefore AZ^2 = AY \cdot AX \) |
| [23] |

**TOTAL MARKS:** [125]