MATHEMATICS P1

NOVEMBER 2019

MARKS: 150

TIME: 3 hours

This question paper consists of 8 pages.
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of NINE questions.

2. Answer ALL the questions.

3. Number the answers correctly according to the numbering system used in this question paper.

4. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.

5. Answers only will not necessarily be awarded full marks.

6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

7. Round off answers to TWO decimal places, unless stated otherwise.

8. Diagrams are NOT necessarily drawn to scale.

9. Write neatly and legibly.
QUESTION 1

1.1 Solve for \( x \) in each of the following:

1.1.1 \( 2x(x-3) = 0 \) \hspace{1cm} (2)

1.1.2 \( 3x^2 - 2x = 4 \) (correct to TWO decimal places) \hspace{1cm} (4)

1.1.3 \( (x-1)(4-x) \geq 0 \) \hspace{1cm} (3)

1.1.4 \( \sqrt{5-x} = x+1 \) \hspace{1cm} (5)

1.2 Solve for \( x \) and \( y \) simultaneously if:

\( x+4 = 2y \) and \( y^2 - xy + 21 = 0 \) \hspace{1cm} (6)

1.3 Discuss the nature of the roots of the equation \( 2(x-3)^2 + 2 = 0 \) \hspace{1cm} (2)

1.4 Determine the value(s) of \( p \) if \( g(x) = -2x^2 - px + 3 \) has a maximum value of \( \frac{3}{8} \). \hspace{1cm} (4) [26]

QUESTION 2

2.1 Simplify fully, WITHOUT using a calculator:

\[ \frac{3^{2x+1} \cdot 15^{2x-3}}{27^{x-1} \cdot 3^{2x-4}} \] \hspace{1cm} (4)

2.2 Solve for \( x \):

2.2.1 \( \left( \frac{1}{2} \right)^x = 32 \) \hspace{1cm} (3)

2.2.2 \( \sqrt[3]{\frac{1}{x^2}} = 4 \) \hspace{1cm} (3)

2.2.3 \( 2^x - \frac{12}{2^x} = -4 \) \hspace{1cm} (5)

2.3 WITHOUT using a calculator, show that \( \frac{\sqrt{2}}{\sqrt{2} + 1} + \frac{4}{\sqrt{2}} \) simplifies to \( 2 + \sqrt{2} \). \hspace{1cm} (5) [20]
QUESTION 3

Given the linear pattern: $-5; 0; 5; ...$

3.1 Determine the general term, $T_n$, of the linear pattern. (2)

3.2 Calculate the value of $T_{12}$. (2)

3.3 Which term in the pattern has a value of 130? (2)

QUESTION 4

4.1 The following number pattern is given: $13; 27; 45; 67; ...$

4.1.1 Is this a quadratic number pattern? Justify your answer with relevant calculations. (2)

4.1.2 Determine the general term, $T_n$, of the quadratic number pattern. (4)

4.1.3 Calculate the value of $T_{100}$. (2)

4.1.4 The first difference between two consecutive terms of the quadratic number pattern is 110. Determine the value of these two terms. (5)

4.1.5 Show that ALL the terms of this quadratic number pattern will be odd numbers. (2)

4.2 $4; x; y; -11$ are the first four terms of a quadratic number pattern.
$2p - 4; p - 3; \frac{p}{2} - 1$ are the first three first differences of the same quadratic number pattern.

Calculate the values of $p$, $x$ and $y$. (5)
QUESTION 5

Given: \( f(x) = \frac{1}{x-3} - \frac{2x+6}{x+3} \)

5.1 Show that \( f(x) \) can be written as \( f(x) = \frac{1}{x-3} - 2 \) \( \) \( (2) \)

5.2 Write down the equations of the asymptotes of \( f \). \( \) \( (2) \)

5.3 Determine the \( x \)-intercept of \( f \). \( \) \( (3) \)

5.4 Determine the \( y \)-intercept of \( f \). \( \) \( (2) \)

5.5 Sketch the graph of \( f \). Show clearly ALL the intercepts with the axes and the asymptotes. \( \) \( (3) \)

5.6 Determine the equation of the axis of symmetry of \( f \) having positive gradient. \( \) \( (3) \)

5.7 The graph of \( f \) is transformed to obtain the graph of \( h(x) = \frac{1}{x} \). Describe the transformation from \( f \) to \( h \). \( \) \( (2) \)

5.8 Write down the domain of \( h \). \( \) \( [19] \)
QUESTION 6

The diagram below shows the graphs of \( f(x) = -x^2 + 2x + 15 \) and \( g(x) = -3x + k \).
Graph \( f \) cuts the \( x \)-axis at \( A(-3; 0) \) and \( B(5; 0) \), the \( y \)-axis at \( C \) and has a turning point at \( D \). Graph \( g \) cuts the \( x \)-axis at \( B \) and the \( y \)-axis at \( C \). \( E \) is a point on \( g \) such \( DE \) is parallel to the \( y \)-axis.

1. Show that \( k = 15 \).
2. Determine the coordinates of \( D \), the turning point of \( f \).
3. Determine the values of \( x \) for which \( f \) is increasing.
4. Calculate the average gradient between points \( A \) and \( D \).
5. Calculate the length of \( DE \).
6. If \( h(x) = f(x-1) - 2 \), determine the equation of \( h \) in the form \( h(x) = a(x + p)^2 + q \).
7. Determine the maximum value of \( p(x) = 3^{f(x)-12} \).
8. Determine the values of \( x \) for which \( f(x) + k = 0 \) will have two distinct real roots.
QUESTION 7

The point \( A(3 ; 54) \) lies on the graph of \( f(x) = 3^{x+p} - 27 \).

7.1 Determine the value of \( p \). \hspace{1cm} (3)

7.2 Determine the range of \( f \). \hspace{1cm} (2)

7.3 Graph \( g \) is obtained by reflecting graph \( f \) about the \( x \)-axis. Determine the coordinates of the \( y \)-intercept of \( g \). \hspace{1cm} (2)

QUESTION 8

8.1 The purchase price of a car five years ago was R200 000. The current book value of the car is R85 000. Using the reducing-balance method of depreciation, calculate the annual rate of depreciation. \hspace{1cm} (3)

8.2 An amount of money was invested at a rate of 8,5% p.a., compounded quarterly. Calculate the effective interest rate per annum of this investment. \hspace{1cm} (3)

8.3 Susan made an initial deposit of R28 000 into an investment account. Three years later she made another deposit of R12 000. She withdrew R6 500 from the account 5 years after the initial deposit was made. The interest rate for the first 4 years was 12% p.a., compounded monthly. Thereafter the interest rate changed to 12,9% p.a., compounded half-yearly.

8.3.1 Calculate how much Susan had in this investment account 2 years after the initial deposit was made. \hspace{1cm} (2)

8.3.2 How much will the investment be worth 8 years after the initial deposit was made? \hspace{1cm} (5)
QUESTION 9

9.1 For any two events, A and B, it is given that \( P(A) = 0.48 \) and \( P(B) = 0.26 \).

Determine:

9.1.1 \( P(A \text{ and } B) \) if A and B are independent events \( \quad (2) \)

9.1.2 \( P(A \text{ or } B) \) if A and B are mutually exclusive events \( \quad (2) \)

9.2 A survey was conducted among 130 Grade 11 learners to establish which snack they prefer to eat while they watch television. The results were summarised in the Venn diagram below. However, some information is missing.

![Venn Diagram](image)

9.2.1 If 29 learners prefer at least two types of snacks, calculate the value of \( x \) and \( y \). \( \quad (4) \)

9.2.2 Determine the probability that a learner who does not eat nuts will either have another snack or no snack while watching television. \( \quad (3) \)

9.3 A group of 200 tourists visited the same restaurant on two consecutive evenings. On both evenings, the tourists could either choose beef (B) or chicken (C) for their main meal. The manager observed that 35% of the tourists chose beef on the first evening and 70% of them chose chicken on the second evening.

9.3.1 Draw a tree diagram to represent the different choices of main meals made on the two evenings. Show on your diagram the probabilities associated with each branch as well as all the possible outcomes of the choices. \( \quad (4) \)

9.3.2 Calculate the number of tourists who chose the same main meal on both evenings \( \quad (3) \)

9.3.3 Show that more tourists opted not to change their choice of main meal during their two visits to the restaurant. \( \quad (2) \)

[20]

TOTAL: 150