NATIONAL
SENIOR CERTIFICATE

GRADE 11

NOVEMBER 2015

MATHEMATICS P1

MARKS: 150

TIME: 3 hours

This question paper consists of 9 pages.
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of ELEVEN questions. Answer ALL the questions.

2. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answer.

3. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

4. Answers only will not necessarily be awarded full marks.

5. If necessary, round off answers to TWO decimal places, unless stated otherwise.

6. Diagrams are NOT necessarily drawn to scale.

7. Number the answers correctly according to the numbering system used in this question paper.

8. Write neatly and legibly.
QUESTION 1

1.1 Solve for \( x \). Round off to TWO decimal places, if necessary.

1.1.1 \( x^2 - 7x + 12 = 0 \) \( (2) \)

1.1.2 \( 6x - 7 = \frac{4}{x} \) \( (4) \)

1.2 Given the following inequalities:

\[ x^2 - 3x \leq 40 \quad \text{and} \quad -4x + 3 < -2 \]

1.2.1 Solve for \( x \) if \( x^2 - 3x \leq 40 \). \( (4) \)

1.2.2 Solve for \( x \) if \( -4x + 3 < -2 \). \( (2) \)

1.2.3 If it is given that \( x \) is a natural number, solve for \( x \) if \( x^2 - 3x \leq 40 \) and \( -4x + 3 < -2 \). \( (2) \)

1.3 Given: \( m + \frac{1}{m} = 3 \)

1.3.1 Determine the value of: \( m^2 - 1 + \frac{1}{m^2} \). \( WOC \)

1.3.2 Hence determine the value of: \( m^3 + \frac{1}{m^3} \). \( (2) \)

[19]

QUESTION 2

2.1 Simplify the following expressions without the use of a calculator.

2.1.1 \( \frac{\sqrt{50} + \sqrt{8}}{7\sqrt{2}} \) \( (3) \)

2.1.2 \( \left[ \frac{16x^{\frac{-5}{6}} - \frac{3}{4}}{81\sqrt{x}} \right] \) \( (4) \)

2.2 Solve for \( x \): \( 27^{x^2 + x} = 3^{3x^2} \times 9 \) \( (3) \)

2.3 If \( 5^{-x} = 10 \), determine the value of \( \frac{2^{x-1} + 2^{x+1}}{5 \times 10^x} \) \( WOC \) \( (5) \)

[15]
QUESTION 3

3.1 Solve for $x$ and $y$ in the following simultaneous equations.

\[ x + y + 2 = 0 \quad \text{and} \quad x^2 + y^2 = 4 \]  \hspace{1cm} (6)

3.2 Determine the nature of the roots of the quadratic equation $ax^2 + bx + c = 0$ if the following conditions are given:

3.2.1 $a < 0$, $b > 0$ and $c = 0$. \hspace{1cm} (2)

3.2.2 $b^2 = ac$ and $b \neq 0$. \hspace{1cm} (2)

3.3 Determine for which value(s) of $p$ will $2x^2 + 4x + 4 - p^2 = 0$ have no real solution. \hspace{1cm} [14]

QUESTION 4

The first term of a linear number pattern is 92 and the constant difference is $-4$.

4.1 Write down the values of the second and third terms of the number pattern. \hspace{1cm} (1)

4.2 Determine an expression for the $n$-th term of the number pattern. \hspace{1cm} (2)

4.3 Determine the value of the eighteenth term. \hspace{1cm} (2)

4.4 If $T_p + T_q = 0$, determine the value of $(p + q)$. \hspace{1cm} [7]

QUESTION 5

5.1 The following number pattern has a constant second difference.

\[ 41; 43; 47; 53; 61; 71; 83; 97; 113; 131; 151; 173; 197; 223; 251; \ldots \]

5.1.1 Write down the value of the constant second difference. \hspace{1cm} (1)

5.1.2 Determine an expression for the $n$-th term of the number pattern. \hspace{1cm} (4)

5.1.3 The first forty terms of the number pattern are all prime numbers. Determine the forty-first term and show that it is not a prime number. \hspace{1cm} (2)

5.1.4 Determine the units digit of the 49 999 998th \((T_{49\,999\,998})\) term. \hspace{1cm} (2)

5.2 The $n$-th term of a number pattern is as follows:

\[ T_n = -5n - 4 \quad \text{if} \quad n \text{ is an even number} \]

and \( T_n = -n^2 + 6 \quad \text{if} \quad n \text{ is an uneven number}. \)

5.2.1 Determine the value of $T_6 + T_7$. \hspace{1cm} (3)

5.2.2 Determine the value of $k$ if $T_k = -219$. \hspace{1cm} [17]
QUESTION 6

6.1 The price of a new school bus is R540 000. The value of the bus decreases at 11% per annum according to the diminishing-balance method. Calculate the value of the bus after 8 years. (2)

6.2 Determine the effective interest rate if an investment earns interest at a nominal interest rate of 11.5% per annum, compounded quarterly. (3)

6.3 Vishnu and Landi receive R15 000 each. They decide to invest the money for a period of 8 years as follows:

Vishnu: Simple interest at 8.7% per annum. At the end of the 8 years Vishnu receives a cash bonus of 3% on the principal amount.

Landi: Interest at 6.9% per annum, compounded monthly.

6.3.1 Calculate the value of Vishnu’s investment after 8 years, including the cash bonus. (3)

6.3.2 Calculate the value of Landi’s investment after 8 years. (3)

6.4 James invests a certain amount for 5 years. The investment earns interest at 12% per annum, compounded monthly, for the full term. James withdraws R2 000 from the account after 18 months. After 5 years the value of the investment is R23 564.

What amount did James initially invest? (5) [16]

QUESTION 7

Given the following two functions:

\[ h(x) = \frac{1}{x} + 5 \quad \text{and} \quad g(x) = x + 5 \]

7.1 Determine the \( x \)-intercept of \( h \). (3)

7.2 Sketch neat graphs of \( h \) and \( g \) on the same set of axes. Clearly show all intercepts with the axes as well as asymptotes. (5)

7.3 Write down the equation of the vertical asymptote of \( h \). (1)

7.4 Determine the coordinates of the points of intersection of \( h \) and \( g \). Show all calculations. (5)

7.5 Write down the equation of \( f \) if \( f \) is the reflection of \( g \) about the line \( y = 4 \). (2)

7.6 Write down the equation if \( h \) is translated so that \((-2; 3)\) is the new point of intersection of the asymptotes. (2) [18]
QUESTION 8

The sketch below shows the graph of \( f(x) = 2 \times a^x - 1 \). The point \( A(1; 5) \) is a point on the graph.

8.1 Show that \( a = 3 \). (2)

8.2 Determine the \( y \)-intercept of \( f \). (2)

8.3 Write down the range of \( f \). (1)

8.4 Determine \( f(0.23) \), rounded off to three decimal places. (2)

8.5 Write down the equation if \( f \) is reflected about the \( x \)-axis followed by a translation of 2 units to the left. (2)
QUESTION 9

The sketch below shows the graph of the function $f(x) = ax^2 + bx + c$.

The straight line with equation $3y = x - 5$ intersects $f$ at $B$. The points $A(-2; 0)$ and $B$ are the $x$-intercepts of $f$. Point $D(-1; 3)$ is a point on $f$.

9.1 Determine the coordinates of $B$. (2)

9.2 Determine the equation of $f$. (4)

9.3 Determine the coordinates of the turning point of $f$. (2)

9.4 Point $E$ is a point on the straight line so that $DE$ is parallel to the $y$-axis. Determine the length of $DE$. (3)

9.5 Calculate the average gradient between $B$ and $D$. (1)

9.6 For which value(s) of $x$ is $x \times f(x) \geq 0$? (2)

[14]
QUESTION 10

A survey regarding their favourite magazine(s) was conducted among 84 high school girls. Three magazines, namely Teen Vogue (T), Drum (D) and People's Magazine (P) were used in the survey.

The results are as follows:
- 41 read Teen Vogue.
- 34 read People's Magazine.
- 40 read Drum.
- 18 read Teen Vogue and Drum.
- 8 read all three magazines.
- 75 read at least one magazine.
- \( n(P \text{ and } D) = 17 \).

The Venn-diagram below shows the above information.

10.1 Determine the values of \( a, b, c, d, e \) and \( f \). (7)

10.2 Determine the probability that a randomly selected girl reads at least two of the three magazines. (2) [9]
QUESTION 11

11.1 The Venn-diagram below shows two independent events, $M$ and $N$.

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S

M
x 0.1

N
0.4

y
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Determine the values of probabilities $x$ and $y$. Show all calculations. (5)

11.2 The following contingency table shows information on the drivers’ tests of 100 drivers tested at a test centre in Port Elizabeth.

<table>
<thead>
<tr>
<th></th>
<th>Male</th>
<th>Female</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pass</td>
<td>30</td>
<td>47</td>
<td>77</td>
</tr>
<tr>
<td>Fail</td>
<td>7</td>
<td>16</td>
<td>23</td>
</tr>
<tr>
<td>Total</td>
<td>37</td>
<td>63</td>
<td>100</td>
</tr>
</tbody>
</table>

A driver is randomly selected from the 100 drivers.

11.2.1 Determine the probability that a female that failed is selected. (2)

11.2.2 Determine the probability that the driver passed, given it is a male. (2)

11.3 William writes a Mathematics examination and an Accounting examination.

He estimates that he has a 40% chance of passing the Mathematics examination. He estimates that he has a 60% chance of passing the Accounting examination. He estimates that he has a 30% chance of passing both.

Determine the probability that William will fail Mathematics and Accounting. (3)

TOTAL: 150