MATHEMATICS P2
EXEMPLARY 2012
MEMORANDUM

MARKS: 100

This memorandum consists of 10 pages.
**NOTE:**
- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum.
- Assuming answers/values in order to solve a problem is NOT acceptable.

**QUESTION 1**

1.1

\[
\text{Mean} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{929}{19} = 48.89
\]

- \(\checkmark \frac{929}{19}\)
- \(\checkmark \text{answer}\)

(2)

1.2

31 ; 31 ; 34 ; 36 ; 37 ; 39 ; 40 ; 43 ; 46 ; 46 ; 48 ; 52 ; 56 ; 60 ; 62 ; 63 ; 65 ; 66 ; 74.

Median = 46

- \(\checkmark \text{arranging in ascending order}\)
- \(\checkmark \text{median}\)

(2)

1.3

Lower quartile = 37

Upper quartile = 62

- \(\checkmark \text{lower quartile}\)
- \(\checkmark \text{upper quartile}\)

(2)

1.4

- \(\checkmark \text{box with median}\)
- \(\checkmark \text{whisker}\)

(2)

[8]
## QUESTION 2

2.1 The modal class is $2500 \leq x < 4500$

2.2

<table>
<thead>
<tr>
<th>Gross Vehicle Mass (GVM) (in kg)</th>
<th>Frequency</th>
<th>Midpoint</th>
<th>Frequency × midpoint</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2500 \leq x &lt; 4500$</td>
<td>103</td>
<td>3500</td>
<td>360 500</td>
</tr>
<tr>
<td>$4500 \leq x &lt; 6500$</td>
<td>19</td>
<td>5500</td>
<td>104 500</td>
</tr>
<tr>
<td>$6500 \leq x &lt; 8500$</td>
<td>70</td>
<td>7500</td>
<td>525 000</td>
</tr>
<tr>
<td>$8500 \leq x &lt; 10500$</td>
<td>77</td>
<td>9500</td>
<td>731 500</td>
</tr>
<tr>
<td>$10500 \leq x &lt; 12500$</td>
<td>85</td>
<td>11500</td>
<td>977 500</td>
</tr>
<tr>
<td>$12500 \leq x &lt; 14500$</td>
<td>99</td>
<td>13500</td>
<td>1 336 500</td>
</tr>
<tr>
<td><strong>Sum</strong></td>
<td><strong>453</strong></td>
<td><strong>13500</strong></td>
<td><strong>4 035 500</strong></td>
</tr>
</tbody>
</table>

Estimated mean $\bar{X} = \frac{403 500}{453} = 8908.39$ kg.

2.3 The estimated mean.
It is more at the centre of the data set. The modal class is found at the extreme left-hand side of the data set.
QUESTION 3

3.1.1

DE = \sqrt{(-3-3)^2 + (3 - (-5))^2}
= \sqrt{100}
= 10
✓ substitution into distance formula
✓ answer (2)

3.1.2

m_{DE} = \frac{-5 - 3}{3 - (-3)}
= \frac{-8}{6}
= \frac{-4}{3}
✓ substitution into gradient formula
✓ answer (2)

3.1.3

m_{EF} = \frac{3}{4}
EF \perp DE

\begin{align*}
-5 - k &= \frac{3}{4} \\
3 - (-1) &= 4 \\
-5 - k &= \frac{3}{4} \\
4 &= 4 \\
-20 - 4k &= 12 \\
-4k &= 32 \\
k &= -8
\end{align*}
✓ m_{EF} = \frac{3}{4}
✓ \frac{-5 - k}{3 - (-1)} = \frac{3}{4}
✓ simplification
✓ k = -8 (4)

3.1.4

M\left(\frac{(-3) + (-1)}{2}; \frac{3 + (-8)}{2}\right)
= \left(-2; \frac{-5}{2}\right)
✓ substitution into midpoint formula
✓ answer (2)
3.1.5 If DEFG is a rectangle then M is also the midpoint of EG.
Let the coordinates of G be \((x ; y)\)
\[
\left(\frac{x + 3}{2} ; \frac{y + (-5)}{2}\right) = \left(-2 ; -\frac{5}{2}\right)
\]
\[
\frac{x + 3}{2} = -2 \quad \text{and} \quad \frac{y - 5}{2} = -\frac{5}{2}
\]
\[
x + 3 = -4 \quad \text{and} \quad y - 5 = -5
\]
\[
x = -7 \quad \text{and} \quad y = 0
\]
\[
\therefore \text{G}( -7 ; 0)
\]

OR

The translation that sends E(3 ; –5) to F(–1; –8) also sends D(–3 ; 3) to G.
\((-1 ; -8) = (3 – 4 ; -5 – 3)\)
\[
\therefore \text{G}( -3 – 4 ; 3 – 3) = (- 7 ; 0)
\]

OR

The translation that sends E(3 ; –5) to D(–3 ; 3) also sends F(–1; –8) to G.
\((- 3 ; 3) = (3 – 6 ; -5 + 8)\)
\[
\therefore \text{G}( -1 – 6 ; -8 + 8) = (- 7 ; 0)
\]

3.2 \[
\sqrt{(x - 1)^2 + (5 - (-2))^2} = \sqrt{53}
\]
\[
(x - 1)^2 + 49 = 53
\]
\[
x^2 - 2x + 1 + 49 - 53 = 0
\]
\[
x^2 - 2x - 3 = 0
\]
\[
(x + 1)(x - 3) = 0
\]
\[
x = -1 \quad \text{or} \quad x = 3
\]
but D is in the second quadrant
\[
\therefore \text{only} \ x = -1 \text{is valid}
\]
### QUESTION 4

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>4.1.1</strong></td>
<td>( \sin C = \frac{AB}{AC} )</td>
<td>✓ AC (1)</td>
</tr>
<tr>
<td><strong>4.1.2</strong></td>
<td>( \cot A = \frac{AB}{BC} )</td>
<td>✓ ( \cot A ) (1)</td>
</tr>
<tr>
<td><strong>4.2</strong></td>
<td>( \sin 60^\circ \cdot \tan 30^\circ ) sec ( 45^\circ )</td>
<td>✓ ✓ substitution</td>
</tr>
<tr>
<td></td>
<td>( \left( \frac{\sqrt{3}}{2} \right) \left( \frac{1}{\sqrt{3}} \right) )</td>
<td>✓ simplification</td>
</tr>
<tr>
<td></td>
<td>( \frac{1}{\sqrt{2}} )</td>
<td>✓ answer (4)</td>
</tr>
<tr>
<td></td>
<td>( \frac{1}{2} \times \frac{1}{\sqrt{2}} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \frac{1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \frac{\sqrt{2}}{4} )</td>
<td></td>
</tr>
<tr>
<td><strong>4.3.1</strong></td>
<td>( r^2 = (-5)^2 + (12)^2 )</td>
<td>✓ ( r^2 = (-5)^2 + (12)^2 )</td>
</tr>
<tr>
<td></td>
<td>( r^2 = 169 )</td>
<td>✓ ( r = 13 )</td>
</tr>
<tr>
<td></td>
<td>( r = 13 )</td>
<td>✓ answer (3)</td>
</tr>
<tr>
<td></td>
<td>( \cos \theta = -\frac{5}{13} )</td>
<td></td>
</tr>
<tr>
<td><strong>4.3.2</strong></td>
<td>( \csc^2 \theta + 1 )</td>
<td>✓ ( = \frac{13}{12} )</td>
</tr>
<tr>
<td></td>
<td>( \left( \frac{13}{12} \right)^2 + 1 )</td>
<td>✓ simplification</td>
</tr>
<tr>
<td></td>
<td>( \frac{169}{144} + \frac{144}{144} )</td>
<td>✓ answer</td>
</tr>
<tr>
<td></td>
<td>( \frac{313}{144} )</td>
<td>(3) [12]</td>
</tr>
</tbody>
</table>
### QUESTION 5

#### 5.1.1

\[ 5 \cos x = 3 \]

\[
\cos x = \frac{3}{5}
\]

\[
x = \cos^{-1} \left( \frac{3}{5} \right)
\]

\[ x = 53,1^\circ \]

\[ \checkmark \cos x = \frac{3}{5} \]

\[ \checkmark \text{answer} \]

#### 5.1.2

\[ \tan 2x = 1,19 \]

\[ 2x = \tan^{-1} (1,19) \]

\[ 2x = 49,95845\ldots^\circ \]

\[ x = 25^\circ \]

\[ \checkmark \text{answer} \]

#### 5.1.3

\[ 4 \sec x - 3 = 5 \]

\[
4 \sec x = 8
\]

\[
\sec x = 2
\]

\[
\frac{1}{\sec x} = \frac{1}{2}
\]

\[
\cos x = \frac{1}{2}
\]

\[
x = \cos^{-1} \left( \frac{1}{2} \right)
\]

\[ x = 60^\circ \]

\[ \checkmark \text{answer} \]

#### 5.2.1

\[ \hat{J}KD = 8^\circ \text{ alternate angles} \]

\[ \checkmark \text{answer} \]

#### 5.2.2

\[ \tan 8^\circ = \frac{5}{DK} \]

\[ DK = \frac{5}{\tan 8^\circ} \]

\[ DK = 35,57684\ldots \text{ km} \]

\[ DK = 35,577 \text{ m} \]

\[ \checkmark \tan 8^\circ = \frac{5}{DK} \]

\[ \checkmark DK = \frac{5}{\tan 8^\circ} \]

\[ \checkmark \text{answer} \]

#### 5.2.3

\[ DS = 35,58 - 8 = 27,58 \text{ km} \]

\[ \checkmark \text{answer} \]

#### 5.2.4

\[ \tan D\hat{S}J = \frac{5}{27,58} \]

\[ D\hat{S}J = \tan^{-1} \left( \frac{5}{27,58} \right) \]

\[ D\hat{S}J = 10,3^\circ \]

\[ \checkmark \tan D\hat{S}J = \frac{5}{27,58} \]

\[ \checkmark \text{answer} \]

[16]
## QUESTION 6

### 6.1.1

- correct $x$-intercepts
- correct $y$-intercept
- asymptotes
- shape (must pass through $(45^\circ; 2)$)

### 6.1.2

- $y = -2 \tan x$

### 6.2.1

- $g(x) = a \sin x$
  - $4 = a \sin 90^\circ$
  - $4 = a(1)$
  - $a = 4$

### 6.2.2

- Range is $-2 \leq y \leq 6$.  
  - $-2$
  - $6$
**QUESTION 7**

| 7.1.1 | \[ AH^2 = 0,8^2 + 1,5^2 \] | ✓ \[ AH^2 = 0,8^2 + 1,5^2 \]  
|       | \[ AH^2 = 2,89 \] | ✓ \[ AH = 1,7 \] (2)  
|       | \[ AH = 1,7 \] |  
| 7.1.2 | Surface area of roof \[ = 4 \times \frac{1}{2} (3 \times 1,7) \] | ✓ \[ 4 \times \frac{1}{2} (3 \times 1,7) \]  
|       | \[ = 10,2 \ m^2 \] | ✓ answer (2)  
| 7.1.3 | Surface area of walls \[ = 4 \times 3 \times 2,1 \] | ✓ \[ 25,2 \ m^2 \]  
|       | \[ = 25,2 \ m^2 \] | ✓ answer (2)  
|       | Total surface area \[ = 10,2 \ m^2 + 25,2 \ m^2 = 35,4 \ m^2 \] |  
| 7.2.1 | Volume \[ = \frac{4}{3} \pi (8)^3 \] | ✓ \[ \frac{4}{3} \pi (8)^3 \]  
|       | \[ = 2144,66 \ mm^3 \] | ✓ answer (2)  
| 7.2.2 | New volume : original volume \[ = 2^3 : 1 \] | ✓ \[ 2^3 \]  
|       | \[ = 8 : 1 \] | ✓ answer (2)  
| 7.2.3 | Volume including silver \[ = \frac{4}{3} \pi (9)^3 \] \[ = 3 \ 053,63 \ mm^3 \] | ✓ \[ \frac{4}{3} \pi (9)^3 \]  
|       | Volume of silver \[ = 3 \ 053,63 - 2144,66 \] \[ = 908,97 \ mm^3 \] | ✓ answer (2)  

**QUESTION 8**

| 8.1 | \[ OQ = 2 \ cm \] | ✓ 2 cm  
|     | .... (the long diagonal of a kite bisects the shorter diagonal) | ✓ correct reason (2)  
| 8.2 | \[ \hat{POQ} = 90^\circ \] | ✓ \[ 90^\circ \]  
|     | .... (the diagonals of a kite intersect at right angles) | ✓ correct reason (2)  
| 8.3 | \[ \hat{QPO} = 20^\circ \] | ✓ \[ \hat{QPO} = 20^\circ \] with correct reason  
|     | .... (the longer diagonal bisects the angles of a kite) |  
|     | \[ \therefore \hat{QPS} = 20^\circ + 20^\circ = 40^\circ \] | ✓ \[ \hat{QPS} = 40^\circ \] (2)  

[6] [12]
QUESTION 9

9.1  O is the midpoint of BD. .... (Diagonals of parm BCDE bisect each other)
F is the midpoint of OE. .... (Diagonals of parm AODE bisect each other)
∴ OF || AB .... (The line joining the midpoints of two sides in a Δ is || to third side)

9.2  AE || OD .... (Opp sides of parm AODE are parallel)
∴ AE || OB
OF || AB .... (proven above)
∴ OE || AB
∴ ABOE is a parallelogram .... (both pairs of opposite sides of quad are parallel)

9.3  In ΔABO and ΔEOD
1. AB = EO ...(Opp sides of parm ABOE are equal)
2. AO = ED ...(Opp sides of parm AODE are equal)
3. BO = DO ...(Diagonals of parm BCDE bisect each other)
∴ ΔABO ≡ ΔEOD (S, S, S)

O is the midpoint of BD
reason –
diagonals of parm
F is the midpoint of OE
reason –
midpoint theorem

AE || OB
reason

OE || AB
reason – opp
sides parallel

AB = EO
AO = ED
reason – opp
sides are equal
BO = DO
reason –
diagonals of parm

TOTAL: 100